Free quarks and antiquarks versus hadronic matter

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Abstract

Meson-meson reactions $A(q_1\bar{q}_1)+B(q_2\bar{q}_2) \to q_1+\bar{q}_1+q_2+\bar{q}_2$ in high-temperature hadronic matter are found to produce an appreciable amount of quarks and antiquarks freely moving in hadronic matter and to establish a new mechanism for deconfinement of quarks and antiquarks in hadronic matter.

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The picture for hadronic matter is that this matter only consists of hadrons and the evolution of this matter is determined by hadron scatterings and hadron flows. Meson-meson elastic scatterings establish thermal states of hadronic matter. At the critical temperature of QCD phase transition quarks and antiquarks automatically move from the small hadron volume toward the large volume of hadronic matter. However, in the present work we show that this pure hadronic matter is mixed with free quarks and antiquarks. The reason is that meson-meson scatterings into free quarks and free antiquarks at high temperature can cause the ratio of free-quark number density to number density of hadronic matter to be larger than 0.1 in a short time period of 0.5 fm/c. We also show that the reactions offer a new way for deconfinement near the critical temperature, and name the deconfinement via the reactions collisional deconfinement. We address the occurrence of deconfinement from variation of number densities of hadrons and free quarks and antiquarks in contrast to the variation of energy density, chiral condensate and the screening mass of a heavy quark-antiquark pair studied by lattice QCD [1]. We focus on the reactions of π, ρ, K and K^* because the four hadron species are dominant meson species in hadronic matter at RHIC [2–4].

The cross section for the meson-meson reaction $A(q_1\bar{q}_1) + B(q_2\bar{q}_2) \to q_1 + \bar{q}_1 + q_2 + \bar{q}_2$ is

$$\begin{aligned}
\sigma &= \frac{(2\pi)^4}{4\sqrt{(P_A\cdot P_B)^2 - m_A^2 m_B^2}} \int \frac{d^3p_{q_1'}}{(2\pi)^3 2E_{q_1'}} \frac{d^3p_{\bar{q}_1'}}{(2\pi)^3 2E_{\bar{q}_1'}} \frac{d^3p_{q_2'}}{(2\pi)^3 2E_{q_2'}} \frac{d^3p_{\bar{q}_2'}}{(2\pi)^3 2E_{\bar{q}_2'}} \\
&\quad \mid \mathcal{M}_{\rm fi} \mid^2 \delta(P_A + P_B - p_{q_1'} - p_{\bar{q}_1'} - p_{q_2'} - p_{\bar{q}_2'}) \\
&= \frac{1}{32(2\pi)^8\sqrt{[s-(m_A+m_B)^2][s-(m_A-m_B)^2]}} \int d\Omega_{q_2'} \frac{d^3p_{q_1'}}{E_{q_1'}} \frac{d^3p_{\bar{q}_1'}}{E_{\bar{q}_1'}} \\
&\quad \frac{\vec{p}_{q_2'}^{\,2} \mid \mathcal{M}_{\rm fi} \mid^2}{\mid\mid \vec{p}_{q_2'} \mid E_{\bar{q}_2'} + (\mid \vec{p}_{q_2'} \mid - \mid \vec{P}_A + \vec{P}_B - \vec{p}_{q_1'} - \vec{p}_{\bar{q}_1'} \mid \cos\Theta) E_{q_2'} \mid}
\end{aligned} \tag{1}$$

where m_A (m_B) and $P_A = (E_A, \vec{P}_A)$ ($P_B = (E_B, \vec{P}_B)$) are the mass and the four-momentum of meson A (B), respectively, and $s = (P_A + P_B)^2$; $p_i = (E_i, \vec{p}_i)$ ($i = q_1', \bar{q}_1', q_2', \bar{q}_2'$) is the four-momentum of a final quark or antiquark, and subscripts of variables for the final quarks and antiquarks are labeled with primes; Θ is the angle between $\vec{p}_{q_2'}$ and $\vec{P}_A + \vec{P}_B - \vec{p}_{q_1'} - \vec{p}_{\bar{q}_1'}$, and $d\Omega_{q_2'}$ is the solid angle centered about the direction of $\vec{p}_{q_2'}$. The transition amplitude $\mathcal{M}_{\rm fi}$ arising from one gluon exchange between one constituent

in meson A and one constituent in meson B is

$$
\begin{aligned}
\mathcal{M}_{\rm fi} &= \sqrt{2E_A 2E_B 2E_{q_1'} 2E_{\bar{q}_1'} 2E_{q_2'} 2E_{\bar{q}_2'}} \\
& \left[V_{q_1\bar{q}_2}(\vec{Q}) \psi_{q_1\bar{q}_1}(\vec{p}_{q_1'\bar{q}_1'} - \frac{m_{\bar{q}_1}}{m_{q_1}+m_{\bar{q}_1}}\vec{Q}) \psi_{q_2\bar{q}_2}(\vec{p}_{q_2'\bar{q}_2'} - \frac{m_{q_2}}{m_{q_2}+m_{\bar{q}_2}}\vec{Q}) \right. \\
& + V_{\bar{q}_1 q_2}(\vec{Q}) \psi_{q_1\bar{q}_1}(\vec{p}_{q_1'\bar{q}_1'} + \frac{m_{q_1}}{m_{q_1}+m_{\bar{q}_1}}\vec{Q}) \psi_{q_2\bar{q}_2}(\vec{p}_{q_2'\bar{q}_2'} + \frac{m_{\bar{q}_2}}{m_{q_2}+m_{\bar{q}_2}}\vec{Q}) \\
& + V_{q_1 q_2}(\vec{Q}) \psi_{q_1\bar{q}_1}(\vec{p}_{q_1'\bar{q}_1'} - \frac{m_{\bar{q}_1}}{m_{q_1}+m_{\bar{q}_1}}\vec{Q}) \psi_{q_2\bar{q}_2}(\vec{p}_{q_2'\bar{q}_2'} + \frac{m_{\bar{q}_2}}{m_{q_2}+m_{\bar{q}_2}}\vec{Q}) \\
& \left. + V_{\bar{q}_1\bar{q}_2}(\vec{Q}) \psi_{q_1\bar{q}_1}(\vec{p}_{q_1'\bar{q}_1'} + \frac{m_{q_1}}{m_{q_1}+m_{\bar{q}_1}}\vec{Q}) \psi_{q_2\bar{q}_2}(\vec{p}_{q_2'\bar{q}_2'} - \frac{m_{q_2}}{m_{q_2}+m_{\bar{q}_2}}\vec{Q}) \right]
\end{aligned}
\tag{2}
$$

where $\vec{Q}$ is the gluon momentum, m_i ($i = q_1, \bar{q}_1, q_2, \bar{q}_2$) is the mass of a constituent quark or antiquark of mesons, and $\vec{p}_{ij}$ is the relative momentum of quark i and antiquark j. For mesons $\psi_{ij}(\vec{p}_{ij})$ is the wave function of the quark-antiquark relative motion in momentum space and satisfies $\int \frac{d^3 p_{ij}}{(2\pi)^3} \psi_{ij}^+(\vec{p}_{ij})\psi_{ij}(\vec{p}_{ij}) = 1$. $\mathcal{M}_{\rm fi}$ is deriveded from the matrix element

$$
\langle q_1, \bar{q}_1, q_2, \bar{q}_2 \mid H_I \mid A, B \rangle = \frac{(2\pi)^3 \delta(\vec{P}_{\rm i} - \vec{P}_{\rm f}) \mathcal{M}_{\rm fi}}{V^3 \sqrt{2E_A 2E_B 2E_{q_1'} 2E_{\bar{q}_1'} 2E_{q_2'} 2E_{\bar{q}_2'}}} \tag{3}
$$

where $\vec{P}_{\rm i}$ ($\vec{P}_{\rm f}$) is the total three-dimensional momentum of the two initial mesons (final quarks and antiquarks); the wave functions of the initial mesons and of the final quarks and antiquarks are normalized to one in volume V, respectively. The interaction is

$$
H_{\rm I} = \tilde{V}(\vec{r}_{q_1\bar{q}_2}) + \tilde{V}(\vec{r}_{\bar{q}_1 q_2}) + \tilde{V}(\vec{r}_{q_1 q_2}) + \tilde{V}(\vec{r}_{\bar{q}_1\bar{q}_2}) \tag{4}
$$

where $\tilde{V}(\vec{r}_{ij})$ is the potential and $\vec{r}_{ij}$ is the relative coordinate of constituents i and j. The potential takes a form [5] whose central interquark potential is obtained from the lattice gauge results of Karsch *et al.* [6]. The potential reflects the medium screening effect at finite temperature. It has been used to calculate temperature-dependent dissociation cross sections of $\pi + J/\psi$, $\pi + \chi_{c1}$ and $\pi + \chi_{c2}$ [7] in the quark-interchange mechanism [8]. Fourier transform of the coordinate-space potential leads to the expression in momentum space

$$
V_{ij}(\vec{Q}) = \frac{\vec{\lambda}_i}{2} \cdot \frac{\vec{\lambda}_j}{2} \left[\frac{4\pi\alpha_s}{\mu^2(T) + \vec{Q}^2} + \frac{6\pi b(T)}{(\mu^2(T) + \vec{Q}^2)^2} - \frac{8\pi\alpha_s}{3m_i m_j} \vec{s}_i \cdot \vec{s}_j \exp\left(-\frac{\vec{Q}^2}{4d^2}\right) \right] \tag{5}
$$

where $\vec{\lambda}_i$, $\vec{s}_i$ and m_i are the Gell-Mann "λ-matrices", the spin and the mass of the constituent i, respectively; $d = 0.897$ GeV [9, 10], $\alpha_s = \frac{12\pi}{25\ln(10+Q^2/X^2)}$ with $X = 0.31$ GeV, $b(T) = b_0[1 - (T/T_c)^2]\theta(T_c - T)$ with $b_0 = 0.35$ GeV^2 and the critical temperature

$T_c = 0.175$ GeV, and $\mu(T) = \mu_0\theta(T_c - T)$ with $\mu_0 = 0.28$ GeV. Determined by meson spectroscopy, the constituent quark masses (CQM) are 0.334 GeV for both the up quark and the down quark and 0.575 GeV for the strange quark [9].

Denote the orbital angular momentum and the spin of meson A by L_A and S_A, respectively. Similar notation is established for meson B. For the three cases: (1) $L_A = L_B = 0$, (2) $L_A = 0$, $L_B \neq 0$, $S_A = 0$, (3) $L_A = 0$, $L_B = 1$, $S_A = 1$, $S_B = 1$, the unpolarized cross section is

$$\sigma^{\rm unpol}_{AB\to {\rm free}}(\sqrt{s}) = \frac{1}{(2S_A+1)(2S_B+1)(2L_B+1)} \sum_{L_{Bz} S S_{q_1'+\bar{q}_1'} S_{q_2'+\bar{q}_2'}} (2S+1)\sigma(L_{Bz}, S, m_S, S_{q_1'+\bar{q}_1'}, S_{q_2'+\bar{q}_2'}, \sqrt{s}) \tag{6}$$

where L_{Bz} is the magnetic quantum number of meson B, the total spin S takes values that are allowed by $\mid S_A - S_B \mid \leq S \leq S_A + S_B$ and m_S is the component of S. $S_{q_1'+\bar{q}_1'}$ ($S_{q_2'+\bar{q}_2'}$) is the total spin of the final constituents q_1 and $\bar{q}_1$ (q_2 and $\bar{q}_2$). The spins S, $S_{q_1'+\bar{q}_1'}$ and $S_{q_2'+\bar{q}_2'}$ satisfy $\mid S_{q_1'+\bar{q}_1'} - S_{q_2'+\bar{q}_2'} \mid \leq S \leq S_{q_1'+\bar{q}_1'} + S_{q_2'+\bar{q}_2'}$. σ is independent of m_S and is calculated at any value subject to the condition $-S \leq m_S \leq S$.

The Schrödinger equation with the temperature-dependent potential $\tilde{V}(\vec{r}_{ij})$ [5] produces temperature-dependent quark-antiquark wave functions which Fourier transform produces momentum-space wave functions used in obtaining the transition amplitude. Cross sections depend on temperature as well as the center-of-mass energy of the two initial mesons. While temperature increases, the confinement potential gets weak and the bound state gets loose. At higher temperature stronger screening leads to larger cross sections for meson-meson reactions.

To uniquely show the role of $A(q_1\bar{q}_1) + B(q_2\bar{q}_2) \to q_1 + \bar{q}_1 + q_2 + \bar{q}_2$, we do not consider any expansion of hadronic matter. The master rate equations for free quarks, pions, rhos, kaons and vector kaons in static hadronic matter are

$$\frac{dn_q}{dt} = \sum_{i=\pi,\rho,K,K^*} \sum_{j=\pi,\rho,K,K^*} \langle v_{\rm rel}\sigma^{\rm unpol}_{ij\to {\rm free}}\rangle n_i n_j \tag{7}$$

$$\frac{dn_\pi}{dt} = -\sum_{j=\pi,\rho,K,K^*} \langle v_{\rm rel}\sigma^{\rm unpol}_{\pi j\to {\rm free}}\rangle n_\pi n_j \tag{8}$$

$$\frac{dn_\rho}{dt} = -\sum_{j=\pi,\rho,K,K^*} \langle v_{\rm rel}\sigma^{\rm unpol}_{\rho j\to {\rm free}}\rangle n_\rho n_j \tag{9}$$

$$\frac{dn_K}{dt} = -\sum_{j=\pi,\rho,K,K^*} \langle v_{\rm rel}\sigma^{\rm unpol}_{K j\to {\rm free}}\rangle n_K n_j \tag{10}$$

$$\frac{dn_{K^*}}{dt} = -\sum_{j=\pi,\rho,K,K^*} \langle v_{\rm rel}\sigma^{\rm unpol}_{K^*j\to {\rm free}}\rangle n_{K^*}n_j \tag{11}$$

where n_q, n_π, n_ρ, n_K and n_{K^*} are the number densities of free quarks, π, ρ, K and K^*, respectively; $v_{\rm rel}$ in the thermal averages denoted by the symbols $< \cdots >$ is the relative velocity of two colliding hadrons. The number density of free antiquarks equals the one of free quarks.

The master rate equations are solved from pure hadronic matter in thermal equilibrium at $t = 0$ fm/c. The initial number densities of free quarks and antiquarks equal zero, respectively, and the number densities of π, ρ, K and K^* are given by $n_j = \int \frac{d^3p_j}{(2\pi)^3}\frac{g_j}{\exp(\sqrt{\vec{p}_j^{\,2}+m_j^2}/T)-1}$ with the degeneracy factor g_j, the hadron mass m_j and $j = \pi, \rho, K, K^*$. The equation solutions as functions of time at different temperatures are shown in Figs. 1-3. When more and more hadrons due to the collisions convert into free quarks and antiquarks, the number density of free quarks increases with increasing time and accordingly the number density of each hadron species decreases. The number densities vary faster at higher temperature because of larger cross sections. We present two solutions for $T = 0.174$ GeV near the critical temperature. The solution shown by the dashed curves is obtained for the final quarks and antiquarks taking the constituent quark masses. The other solution shown by the solid curves corresponds to the chiral limit (CL) of final quarks and antiquarks. A comparison of the five dashed curves shows that the number density of free quarks exceeds the number densities of π, ρ, K and K^* at $t = 0.83$ fm/c, 0.21 fm/c, 0.33 fm/c and 0.18 fm/c, respectively. In the chiral limit the number density of free quarks exceeds the number densities of π, ρ, K and K^* at $t = 0.28$ fm/c, 0.1 fm/c, 0.13 fm/c and 0.08 fm/c, respectively. Therefore, while the chiral symmetry is restored, hadrons convert quickly into free quarks and antiquarks.

Table 1: Ratios of number densities at $t = 0.5$ fm/c.

T (GeV)	n_q/n_π	n_q/n_ρ	n_q/n_K	n_q/n_{K^*}	$n_q/(n_\pi+n_\rho+n_K+n_{K^*})$
0.174 (CL)	1.83	5.71	3.75	6.33	0.87
0.174 (CQM)	0.63	2.60	1.50	2.86	0.33
0.16	0.19	1.01	0.53	1.23	0.11
0.15	0.08	0.46	0.23	0.62	0.05
0.14	0.03	0.20	0.10	0.29	0.02

The ratio of the free-quark number density to the pion number density etc. are listed in Table 1. The number density of hadronic matter is approximately equal to $n_\pi + n_\rho + n_K + n_{K^*}$. The first and second rows are obtained in the chiral limit and in the use

of the constituent quark masses, respectively. In such a short time period of 0.5 fm/c a large amount of quarks are produced near the critical temperature by $A(q_1\bar{q}_1)+B(q_2\bar{q}_2) \to q_1+\bar{q}_1+q_2+\bar{q}_2$ so that the free quarks are very important in hadronic matter. At $T = 0.16$ GeV an appreciable amount of free quarks exist in hadronic matter. When temperature is lower than 0.14 GeV, the ratio $n_q/(n_\pi + n_\rho + n_K + n_{K^*})$ is so small that free quarks can be neglected. This can also be recognized by the flat long dashed lines in Figs. 1-3.

At $T = 0.174$ GeV, if all of π, ρ, K and K^* break up into free quarks and antiquarks, the number density of free quarks is 0.40 fm^{-3}. In the chiral limit about 75% of the four species of hadrons dissociate at $t = 1.7$ fm/c and 90% at $t = 4.7$ fm/c. While the constituent quark masses are used, about 70% of the hadrons dissociate at $t = 5.0$ fm/c and 90% at $t = 16.0$ fm/c. We have seen now that the deconfinement process of hadronic matter in the chiral limit is much faster than in the use of the constituent quark masses.

As a first step that we study the role of $A(q_1\bar{q}_1)+B(q_2\bar{q}_2) \to q_1+\bar{q}_1+q_2+\bar{q}_2$, we haven't considered quark-antiquark annihilation process. If the contribution of the annihilation process was included, more quarks and antiquarks would be produced.

In conclusion, the reaction $A(q_1\bar{q}_1) + B(q_2\bar{q}_2) \to q_1 + \bar{q}_1 + q_2 + \bar{q}_2$ gets important in hadronic matter at high temperature because of the medium screening effect. Due to the reactions of π, ρ, K and K^*, hadronic matter is mixed with free quarks and antiquarks. The reaction is a new dynamical process for the deconfinement transition of hadronic matter.

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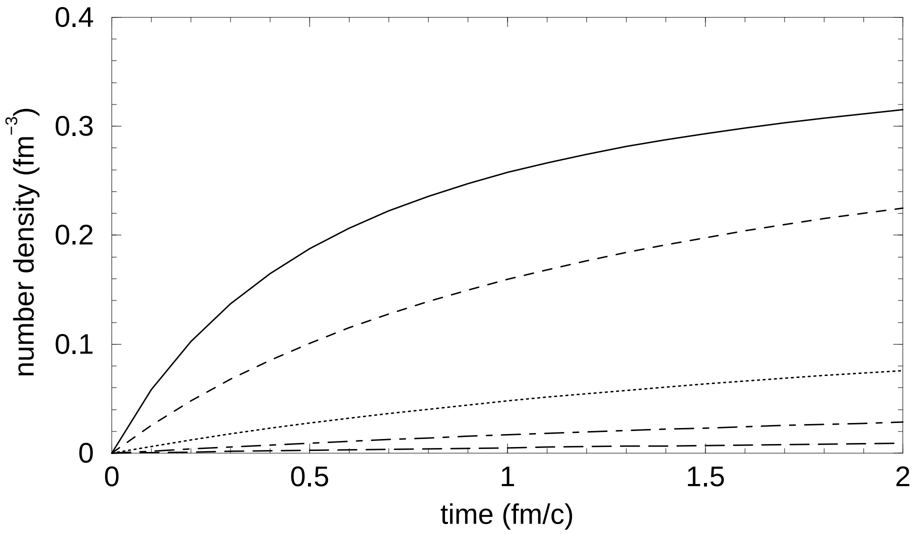

Figure 1: Number densities of free quarks or antiquarks with the constituent quark masses at $T = 0.14$ GeV (long dashed), 0.15 GeV (dot-dashed), 0.16 (dotted), 0.174 GeV (dashed). The solid curve denotes the number density in the chiral limit at $T = 0.174$ GeV.

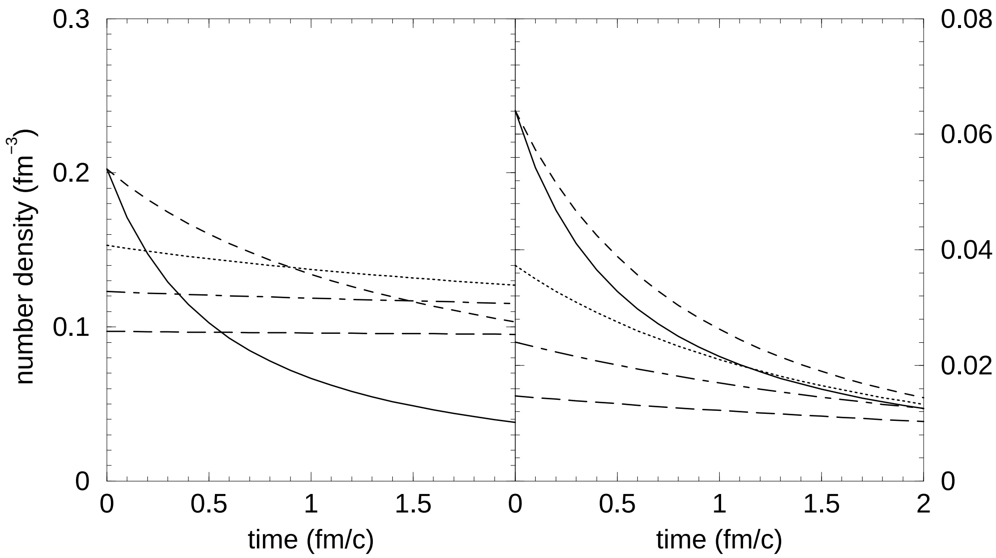

Figure 2: The same as Fig. 1 except for pions in the left panel and rhos in the right panel.

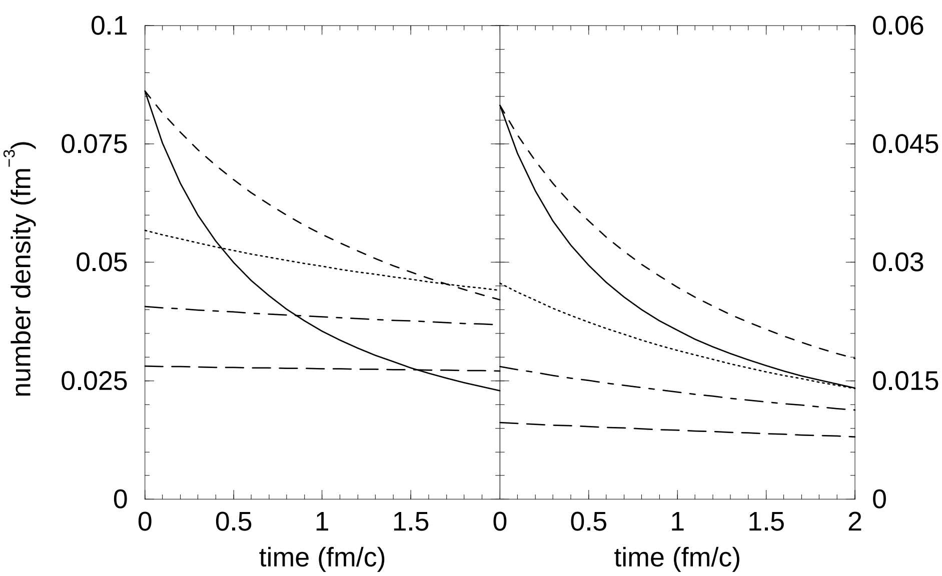

Figure 3: The same as Fig. 1 except for kaons in the left panel and vector kaons in the right panel.